phys. stat. sol. (b) **218**, 251 (2000)

Subject classification: 72.20.Ee

Current and Voltage Distributions in Resistor Network with a Broad Distribution of Conductances

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(Received September 7, 1999)

Random resistor networks with bonds occupied randomly by conductances $g = \exp(-\lambda x)$, where x is the random variable on [0, 1] and $\lambda \gg 1$ are investigated by means of numerical simulations. The problem of correlation between local conductance and local current or local voltage is addressed. The distributions of currents, voltages and power dissipated are calculated for separate subsets of bonds with identical value of conductance. It occurs that subsets of highly/poorly conductive bonds have identical distributions of currents/voltages. From this we conclude that within the subset of highly/poorly conducting bonds local conductance and local current/voltage are statistically independent variables.

Random resistor network is a powerful tool of modelling conduction processes in semiconductors, thin and thick films. In this approach the physical structure is mapped onto an electrical network. When dc features are considered the branches of the network become resistors which take on random values of conductance g. Its distribution depends on the structure being modelled. In many physical cases, for example in case of Variable Range Hopping (VRH) in amorphous semiconductors [1], thin films or granular metals and semiconductors, this distribution is very broad [2]. Namely, $g = \exp(-\lambda x)$, where x is a random variable uniformly distributed between 0 and 1 and $\lambda \gg 1$. Transport properties of such network are then described by the distributions of local currents and voltages. For example, the second and the fourth moments of these distributions are related to the overall conductance G and its fluctuations, respectively. In this paper, the distributions of local currents and voltages in random resistor network with (exponentially) broad distribution of bond conductances are considered. This problem has been addressed already by Roux et al. [3]. They have studied it both theoretically and by means of numerical simulations of a 2D network. One of the most important questions they answered was about the correlation between local conductance g and current j flowing at the same point. They have found that although some correlations between local conductance and local power dissipation e do exist they are rather small and can be neglected. The conclusion was then that g and $e = j^2/g$ are *statistically independent* variables. In this paper, we present new results derived on the basis of numerical simulations of 3D network. In view of these results the conjecture about the statistical independence of e and g is not confirmed. We have found that the distribution of local power dissipation depends on the bond conductance. This can be seen in Fig. 1, where the map of *joint* power–conductance distribution $n(\ln e, x)$ is shown. The front (high power part) of this distribution follows the line

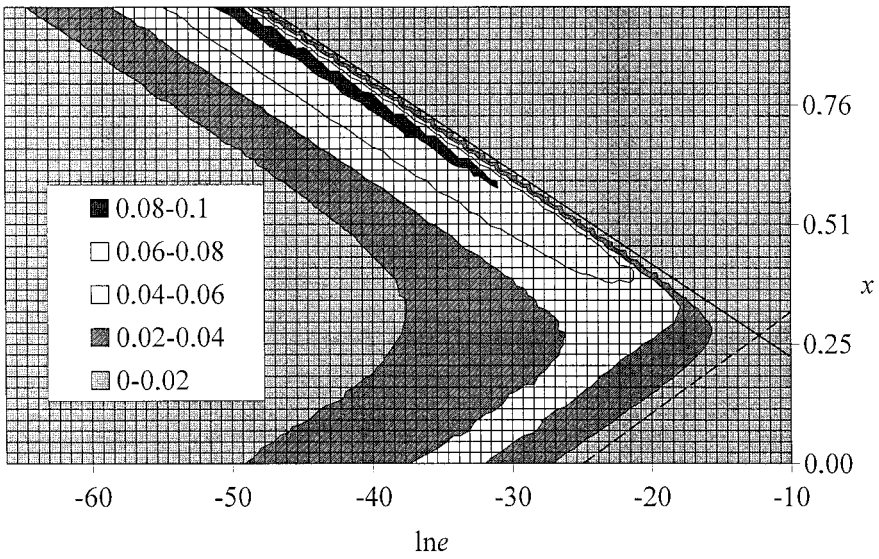

Fig. 1. The map of *joint* distribution $n(\ln e, x)$ versus the logarithm of local power dissipation, $\ln e$, and scaled logarithm of local conductance, $x = -\ln g/\lambda$ obtained for resistor network biased by unit voltage. The network was the simple cubic lattice of size $L = 11$ filled with conductances $g = \exp(-\lambda x)$, with $\lambda = 50$. The lines are drawn according to the equations $\ln e = -\lambda x$ (solid ones) and $\ln e = \lambda(x-0.5)$ (dashed ones)

$\ln e = -\lambda x$ in the region $x > 0.25$ and the line $\ln e = \lambda(x - 0.5)$ in the region $x < 0.25$. This behaviour can be explained within the framework of critical path analysis [1]. The idea is to treat all the bonds with conductance $g > g_c$ as conducting bonds whereas those with $g < g_c$ as insulating ones. To form a percolating cluster the concentration of conducting bonds must be equal to the percolation threshold x_c, which for simple cubic lattice is $x_c \approx 0.25$. Consequently, $g_c \equiv \exp(-\lambda x_c)$. g_c is the smallest conductance in the percolation cluster which determines the overall network conductance, $G \cong g_c$. When unit voltage $V = 1$ biases the network then the overall current is $J = VG \cong g_c$. The maximum voltage drop that could appear across any bond is $v = V = 1$. The maximum local energy dissipation is then $e = V^2 g$. This means that the distribution $n(\ln e, x)$ is bounded by the line: $\ln e = \ln g = -\lambda x$ On the other hand, the maximum current that may flow through a bond is J. The maximum power dissipated in a bond is then, $e = J^2/g$, and so the distribution $n(\ln e, x)$ is bounded also by the line $\ln e = -2\lambda x_c + \lambda x \cong \lambda (x - 0.5)$. However, since current J flows only through the bonds that form percolating cluster (i.e. those with $x < x_c$) the distribution $n(\ln e, x)$ achieves the latter bound only in the region $x < x_c$.

Let us now address the problem of correlations between local conductance g and voltage drop v across it. The quantity we should consider is the normalized voltage, $v = v/V$. In Fig. 2 the (conditional) distributions $n_v(\ln v^2/\lambda, x)$ for x fixed are drawn versus $\ln v^2/\lambda$ for several values of x. For $x > x_c$ the distributions collapse giving rise to the conclusion that in this region of x the correlation between local conductance and voltage drop across it does not exist. As x decreases the shape of $n_v(\ln v^2/\lambda, x)$ becomes x dependent (not shown in Fig. 2) and this means that such correlation exists for $x \leq x_c$

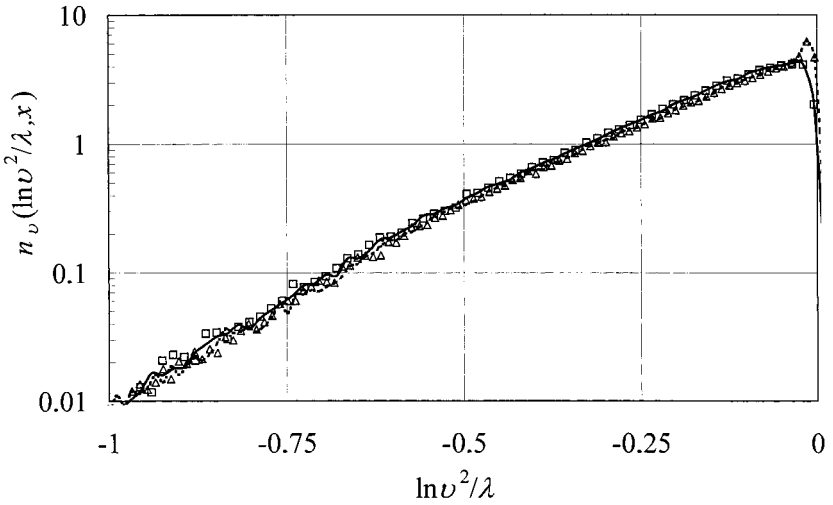

Fig. 2. Distributions $n_v(\ln v^2/\lambda, x)$ as a function of scaled normalized voltage $\ln v^2/\lambda$ obtained for various values of x and λ: $x = 0.97$, $\lambda = 70$ (dashed line), $x = 0.97$, $\lambda = 50$ (solid line), $x = 0.81$, $\lambda = 70$ (triangles), $x = 0.74$, $\lambda = 50$ (squares). The size of the network (simple cubic lattice) was $L = 11$

($g \geq g_c$). Let us now comment on the choice of the scaling variable on the horizontal axis in Fig. 2. We have concluded already that maximum voltage that could appear on conductance g is $v = V$ ($v = 1$). The bonds with $v = 1$ form the front of the distribution $n(\ln e, x)$ in Fig. 1 in the region $x > x_c$. The tail of this distribution is formed by the bonds with minimum voltage. Such voltage appears on the bonds with the largest conductance, i.e. those with $x = 0$. By definition, bonds with $x = 0$ belong to the percolation cluster and current flowing through them is of the order of $J \cong g_c$. Thus, the minimum voltage $v_{\min} = J/g(x = 0) = J$ is of the order of J and the tail of the distribution $n(\ln e, x)$ in the region $x > x_c$ is bounded by the line $\ln e = \ln(gv_{\min}^2) = -\lambda x - 2\lambda x_c$. Now we can estimate the width of the distribution $n(\ln e, x)$ for fixed x as $w \sim 2\lambda x_c$. This is also the width of the distribution of normalized voltage $\ln v^2$. In order to get collapsing of the distributions for different λ values we should then use the scaled variable: $\ln v^2/\lambda$. This is confirmed in Fig. 2 where the distributions n_v obtained for $\lambda = 50$ and 70 are shown to collapse.

Eventually, let us consider the correlation between local conductance g and current j at the same point. We consider the normalized current $i = j/J$. In Fig. 3 the (conditional) distributions $n_i(\ln i^2/\lambda, x)$ with fixed x are drawn versus $\ln i^2/\lambda$ for several values of x and for $\lambda = 50$ and 70. This time the collapse of the distributions is excellent for $x < x_c$. This means that a current which flows through a highly conducting bond (small x) does not depend on its conductance. In other words, i and x are statistically independent variables in the region $x < x_c$. This conclusion does not hold for $x \geq x_c$ where the shape of $n_i(\ln i^2/\lambda, x)$ becomes x dependent (not shown in Fig. 3). One may wonder that current distributions in Fig. 3 extend onto the region $i > 1$, what by definition is forbidden. This inconsistency comes from the method used in numerical simulations. We have used the unit voltage to bias the network and counted the number of bonds carrying a given current j irrespective of the total current J which flows through the network. The nor-

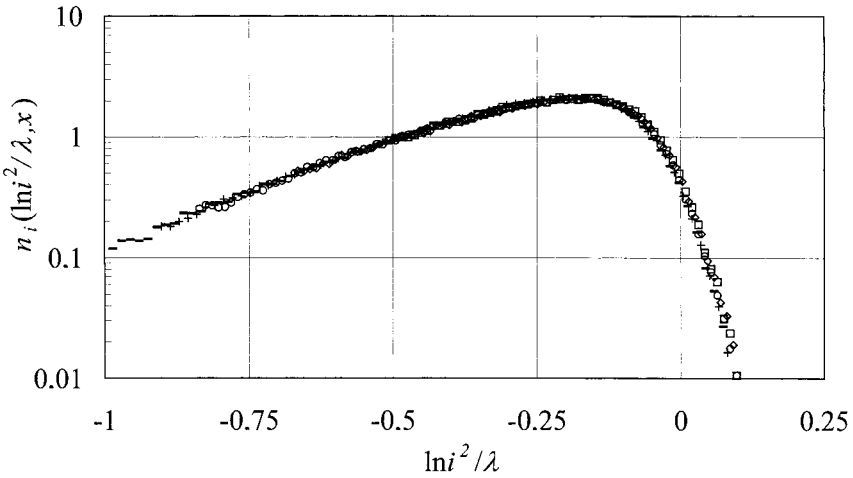

Fig. 3. Distributions $n_i(\ln i^2/\lambda, x)$ of scaled normalized currents $\ln i^2/\lambda$ calculated for various values of x and λ: $x = 0$, $\lambda = 70$ (squares), $x = 0.08$, $\lambda = 70$ (diamonds), $x = 0.16$, $\lambda = 70$ (circles), $x = 0$, $\lambda = 50$ (plus sign), $x = 0.12$, $\lambda = 50$ (minus sign)

malized current i was then computed as $j/\langle J\rangle$ instead of as j/J. Thus, the distribution displayed in Fig. 3 in fact is the distribution of the quantity $\ln i^2/\lambda + 2\ln(J/\langle J\rangle)/\lambda$. For finite L the percolation threshold x_c is the random variable with a distribution, which is peaked around $\langle x_c\rangle \approx 0.25$ but has some non-negligible width [4]. This makes the term $2\ln(J/\langle J\rangle)/\lambda = 2(x_c - \langle x_c\rangle)$ also a random variable whose distribution convolutes with that of $\ln i^2/\lambda$ to give the distribution in Fig. 3. This effect, although influences the shape of the distribution in Fig. 3, does not challenge our major conclusion about the lack of correlation between the local values of i and g for highly conducting bonds.

In conclusion, the problem of correlations between the local conductance and local current and voltages in random resistor networks with broadly distributed bond conductances has been studied. It was shown that within the subset of for highly/poorly conducting bonds local conductance and local current/voltage are statistically independent variables.

Acknowledgement The work was supported by TURz grants No. U 5237/BW and U5232/DS.

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