

Conductance Distribution in Superlocalization Regime

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The phenomenon of superlocalization has been studied by means of numerical simulations performed on two-dimensional percolation cluster at the threshold site concentration $p = p_c = 0.593$. It is shown that superlocalization takes place also for the states inside the band. In this case the averaged logarithm of conductance scales with the size of percolation cluster L as $\langle \ln g \rangle \sim -L^{d_\phi}$ with localization exponent $d_\phi \cong 1.14$. The distribution of $\ln g$ arises from the multiplication of two random variables, namely the chemical length l and inverse localization length γ . The distribution of γ is Gaussian with size independent average and variance which scales like $\text{var } \gamma \sim \langle l \rangle^{-1}$. The distribution of l has a long tail towards large l . Its average and variance scale like $\langle l \rangle \sim L^{d_\phi}$ and $\text{var } l \sim \langle l^2 \rangle$. There is a small correlation between l and γ , which decreases as $L^{d_\phi/2}$ as the size of the percolation cluster increases.

It is well known that in fractal geometry electronic wave functions are superlocalized. Namely, their mean amplitudes behave like [1, 2]

$$|\psi(r)| \sim \exp(-r/\xi r)^{d_\phi}, \quad (1)$$

where r is the distance from the center of the wave function ψ , ξr is the localization length measured in r -space and d_ϕ is the localization exponent. This was rigorously proven for deep states below the band for which the relation $d_\phi = \zeta_1$, where ζ_1 is the exponent which describes scaling of the average chemical length (the shortest part) of the percolation cluster is expected [2]. Namely, $\langle l \rangle \sim L^{\zeta_1}$. Less knowledge concerns the states within the band. Here by the theory only the inequality $1 \leq d_\phi \leq \zeta_1$ holds [2, 3] whereas numerical simulations give incompatible results [4]. Our recent finite size scaling calculations of quantum mechanical conductance show that superlocalization takes place also for the states inside the band [5]. We have found that for 2D percolation cluster the geometrical average of $g \sim |\psi|^2$ scales like in Eq. (1) and exponent $d_\phi \cong 1.14$ was found for energies $E = -3.5t$ and $0.5t$, where t is the hopping matrix element (see Fig. 1). In this paper we focus on the distribution of conductance in the superlocalization regime. As in the previous work [5] our main tool is the use of numerical simulations. The simulations were performed on 2D percolation cluster at critical sites concentration $p = p_c = 0.593$ by the finite size scaling technique. The conductance g was calculated with the help of Landauer-Büttiker formalism and Green's function technique for increasing size L of square lattice (see [5] for more details of the computation technique). The population of the samples was very large (50000) so the evaluation of conductance distribution was possible (see Fig. 2a). Unlike for Euclidean geometries the histogram of $z \equiv -\ln(g/g_0)/2$ is not Gaussian: a long tail grows up for small g 's (large z 's).

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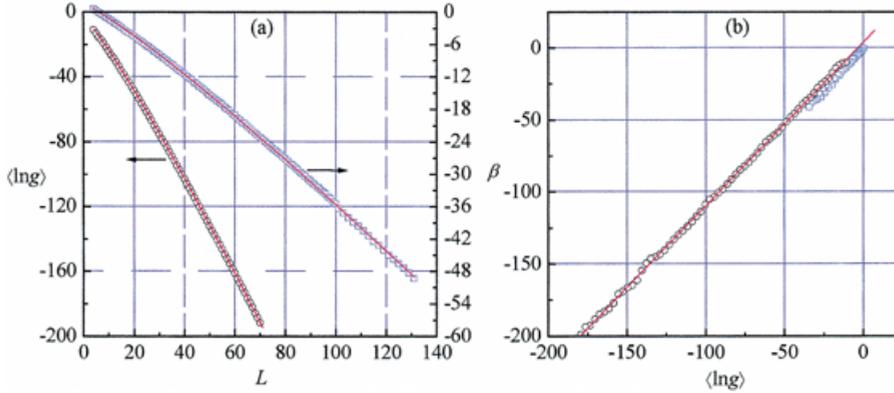


Fig. 1 (online colour). a) Logarithm of conductance g averaged over 50000 configurations of 2D percolation cluster of size L . Data are for $E = -3.5t$ (circles) and $E = 0.5t$ (squares). Lines are the plots of $\langle \ln g \rangle = \ln g_0 - 2\gamma L^{d_\phi}$ with $\ln g_0 = -2.9$, $\gamma = 0.78$, $d_\phi = 1.144$, for $E = -3.5t$ and $\ln g_0 = 1.15$, $\gamma = 0.087$, $d_\phi = 1.138$, for $E = 0.5t$. b) Function $\beta \equiv \frac{\partial \langle \ln g \rangle}{\partial \ln L}$ versus $\langle \ln g \rangle$ for the data from a). The slope of the line is 1.135

The problem of the distribution of wave functions amplitudes on percolation clusters has been already addressed in a number of papers [6–8]. It was concluded that amplitudes $|\psi|$ are distributed log-normally in l -space [7]. The long tail observed in L -space results then from the integration over chemical lengths which (even for constant L) are broadly distributed according to [9]

$$\phi(l | L) \equiv \phi(l | r) = \frac{C_1}{l} \left[\frac{L}{l^{1/\xi_1}} \right]^b \exp \left[-C_2 \left[\frac{L}{l^{1/\xi_1}} \right]^\delta \right]. \quad (2)$$

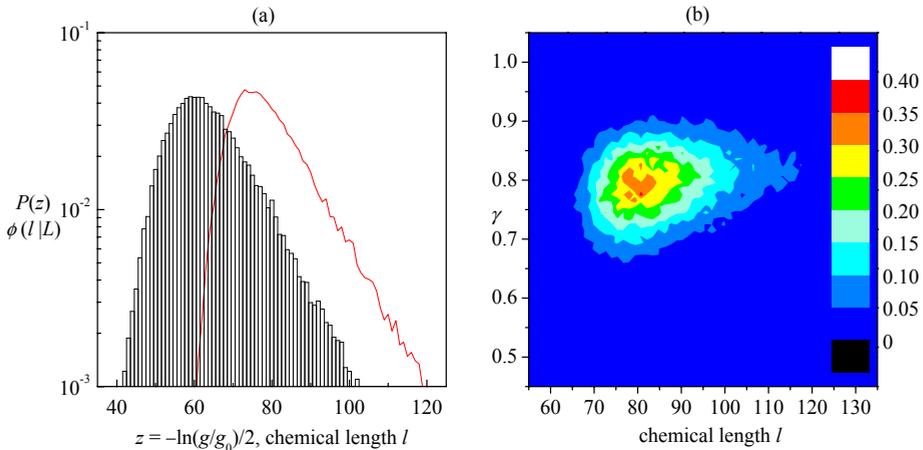


Fig. 2 (online colour). a) Distribution of log conductance rescaled to $z = -\ln(g/g_0)/2$ (bars) in order to make connection with the distribution $\phi(l | L)$ of chemical length l (line) b) (colour) The map of joint distribution $\Pi(l, \gamma)$ of chemical length l and inverse localization length $\gamma = 1/\xi_1 = z/l$. Data in a) and b) are for $L = 50$

In this approach it was assumed that $z = -\ln |\psi/\psi_0|$ is the product of two random variables, $z = \gamma l$, namely the chemical length l and inverse localization length measured along the chemical path (the first Lyapunov exponent) $\gamma = 1/\xi_l$. It was also assumed that l and γ are statistically independent. In this case one has $\langle z \rangle = \langle \gamma \rangle \langle l \rangle$ and the scaling $\langle \ln(g/g_0) \rangle = -2(r/\xi_r)^{d_\phi}$ was recovered.

In the following we address the problem of correlation between l and γ . In case such correlation exists the distribution of z should be expressed in terms of joint chemical length, inverse localization length distribution $\Pi(l, \gamma)$

$$P(z | L) = \int \Pi(l, \gamma | L) \frac{1}{l} dl. \tag{3}$$

A sample map of $\Pi(l, \gamma)$ calculated in our simulations for $L = 50$ is shown in Fig. 2b²). One can see that $\Pi(l, \gamma)$ can hardly be written as $\Pi(l, \gamma) = \phi(l | L) N(\gamma | L)$. It means that small correlation between l and γ exists. The measure of this correlation is a covariance, $\text{cov}(\gamma, l)$, of approximately 0.15. When the size L of the percolation cluster increases this covariance increases rather slowly (see Fig. 3b). This is in contrast to the variances of l and γ , which both show up a strong L dependence. The former increases like $\text{var } l \sim L^{2d_\phi} \sim \langle l \rangle^2$, in agreement with Eq. (2). The latter decreases as $\text{var } \gamma \sim L^{-d_\phi} \sim \langle l \rangle^{-1}$ (see Fig. 3a). This means that the correlation between l and γ decreases as size L of the percolation cluster increases. This conclusion holds at least in the studied range of L 's. Of great interest are not only variances but also the whole distributions of l and γ . They can be obtained as marginal distributions of $\Pi(l, \gamma)$. In agreement with statistics of Lyapunov exponents in the localized regime [10] the distribution $N(\gamma | L)$ is Gaussian (see Fig. 4b). Its average almost does not depend on size of the percolation cluster (see Fig. 3b). This is unlike the distribution $\phi(l | L)$ for which we have found that it is well fitted by Eq. (2). The exponents $b = 12.4$ and $\delta = 8.7$, we have found, differ from those of Refs. [8, 9] due to different geometries considered in these references and ourselves. They consider the distribution $\phi(l | r)$ between two points on the percolation cluster separated by distance r while our distribution is for two parallel lines (electrodes) (distance L apart) intersecting the percolation cluster.

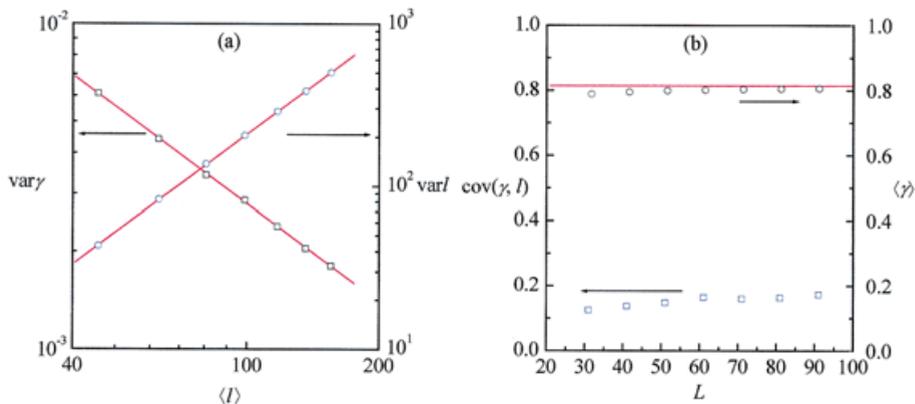


Fig. 3 (online colour). a) Variances of l and γ as a function of average chemical length $\langle l \rangle$. Lines have the slopes of -1 and 2 b) Covariance between l and γ and average inverse localization length $\langle \gamma \rangle$ as a function of size L . Solid line indicates the value 0.813 of the first Lyapunov exponent as $L \rightarrow \infty$

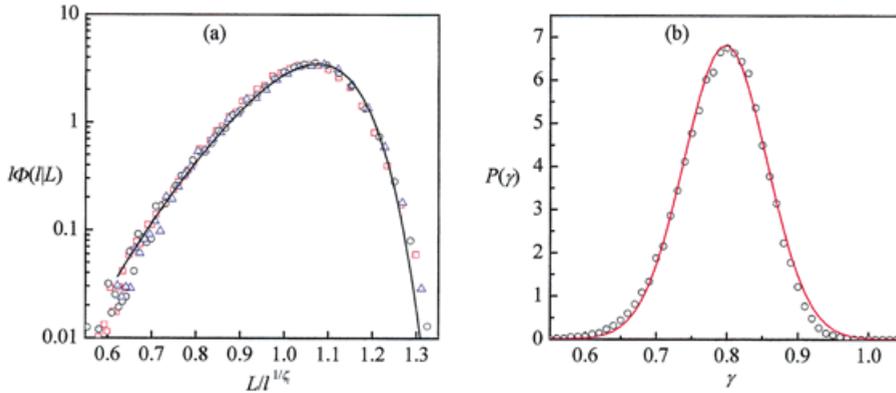


Fig. 4 (online colour). a) Test of scaling of the distribution of chemical length l . Symbols refer to different size $L = 30$ (squares) 50 (circles) 100 (triangles). Solid line is the plot of Eq. (2). b) Distribution of inverse localization length γ for $L = 50$

In conclusion we have shown that distribution $P(z)$ of log conductance, $z = -\ln(g/g_0)/2$, in the superlocalization regime arises from the “convolution” of the distribution $\phi(l|L)$ of chemical distance l and the distribution $N(\gamma|L)$ of the first Lyapunov exponent $\gamma = 1/\xi_l$. For $N(\gamma|L)$ we have found that it is Gaussian with almost L independent average $\langle\gamma\rangle$ and variance which scales like $\text{var } \gamma = \langle l \rangle^{-1}$. There is a small correlation between l and γ that decreases as L increases. In the limit of large L the variance of $\ln g$ arises directly from the variance of l , $\text{var}(\ln g) \sim L^{2d_\phi}$. Starting from this zero-temperature conductance fluctuations we can obtain the behavior at low temperatures by replacing the geometrical size by a “phase coherence length” L_ϕ . In the superlocalized regime the latter is given by the hopping distance $r_h \sim T^{-1/(D+d_\phi)}$, where D is the fractal dimension of the percolation cluster [1–3]. It is now obvious that fluctuations $\delta \ln g \equiv \sqrt{\text{var}(\ln g)}$ diverges as $T \rightarrow 0$ according to

$$\delta \ln g \sim T^{-d_\phi/(D+d_\phi)}. \quad (4)$$

Since the average of $\ln g$ diverges exactly in the same way [1–3] this result implies that relative fluctuations of $\ln g$ does not vanish as $T \rightarrow 0$. This result should be contrasted with ordinary Mott VRH, for which $\delta \ln g / \langle \ln g \rangle$ was found to vanish as $T^{1/2(1+d)}$ [11]. Obviously this difference should be observed only in higher dimensions since for 1D there is no dispersion of chemical length at all.

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